Towards A Possible Charge-Kondo Effect in Optical Lattices

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The Kondo effect underpins a large body of recent developments in the physics of d- and f-band compounds. Although its *charge* analog is a rarity in solids, the recent observations of the charge Kondo effect and the consequent rise in superconducting T_c encourage a search for other accessible systems. Motivated by the possibility of wilfully tuning the *sign* of the inter-electronic interaction in optical lattices, we study conditions for the elusive *charge Kondo liquid* (CKL) state to manifest. We propose that a combination of Feshbach resonances and sequentially controlled laser pulses may produce the CKL. We show that the CKL is *never* a stable ground state, appearing only when the ordered ground states are destabilized. Finally, we discuss interesting analogies with nuclear matter.

The Kondo effect has historically played a monumental role in solid state physics [1], where quasi-bound state (singlet) formation involving a local moment screened by the itinerant electron spins leads a renormalized Fermi liquid (FL) with huge enhancements of quasiparticle masses, whence the name "heavy fermion". Subsequently, more exotic, quadrupolar [2] and orbital [3] versions were invoked in other contexts. The destruction of FL behavior at quantum phase transitions near magnetic order in some f-electron systems is thought to be linked to "unbinding" of this Kondo quasi-bound singlet state [4]. However, the intriguing possibility of observing a *charge* analogue of the Kondo effect, requiring an *attractive*, local interaction, has not been addressed carefully. In practice, materials such as $Ba_{1-x}(K, Pb)_x BiO_3$ are modelled by variants of $U < 0$ Hubbard model [5, 6, 7, 8].

Recent experimental realization [9] of the *charge* Kondo effect (CKE), predicted theoretically some time back [10], and the consequent rise of superconducting T_c in doped $PbTe$ [11] and $SnTe$ [9], has re-ignited interest in the $U < 0$ models. Here, chemical doping ($\leq 2\%$) generates a narrow impurity band in the semiconductor band gap, facilitating use of a $U < 0$ lattice models in the intermediate-to-strong coupling limit ($|U|/W_{imp} > 1$). The interesting issues for theory are thus: *What are the precise conditions under which a charge-Kondo Fermi liquid state (the CKL state) can appear? Is it ever a stable phase at zero temperature? Might a quantum phase transition (QPT), reminiscent of its well-studied spin analogue, occur, and if so, how? What instabilities might one generically expect to arise near such a QPT?*

Advances in artificially engineered fermionic/bosonic optical lattices allowing wilful manipulation of parameters open up yet another route to test model Hamiltonian predictions. Such systems may hold more promise for CKE, given the paucity of real materials exhibiting $U < 0$. In contrast to real materials, these systems are free of inhomogeneities. In this context, various interacting models have already been realized [12, 13]. The physical conditions for this lie well within the regime of experimental techniques available. In practice, different hyperfine states of same or different atomic species (acting, as it may, as different fermionic/bosonic species) can be trapped and controlled independently. A particularly attractive feature is their ready tunability: it is even possible to choose the sign of the interaction in such systems; a mixture of two fermionic species interacting via an attractive interaction is achieved by forcing a mixture of two such atomic spin states through a Feshbach resonance, whence a bound state appears in the two particle problem. A p-wave Feshbach resonance [14] can even create a *tunable* asymmetry in the interactions, allowing one to access more complex "hidden" ordered states. Further, the periodic potential of each species of atoms was independently tuned in an optical lattice [15]. This could facilitate observation of the band insulator (driven by staggered periodic lattice potential) to Mott insulator (due to on-site interaction, U) transition, and possible emergence of correlated metallic phases sandwiched between them.

Optical lattices thus provide a unique tool to address interesting questions posed above. A suitable model for a two-component fermionic system (having an attractive two-body interaction), *with* temporally separated laser pulses simulating inter-site *hybridization* between the two spinless fermionic species (a, b) is described by the Hamiltonian

$$H = \sum_{<i,j>} t_{ab}(a_i^\dagger b_j + h.c.) - U_{ab} \sum_i n_{ia} n_{ib} + \Delta_0 \sum_i (n_{ia} - n_{ib}) + \sum_{i,\alpha=a,b} \epsilon_\alpha n_{i\alpha} \qquad (1)$$

The hoppings are t_{aa} (t_{bb}) for $a-a$ ($b-b$) hopping and t_{ab} for $a-b$ hopping between nearest neighbors. Δ is the "charge transfer" term related to on-site fermionic energies. In typical optical lattice systems, the model parameters expressed in terms of the recoil energy are, hopping $t = \frac{4E_R}{\sqrt{\pi}}(\frac{V_0}{E_R})^{3/4} e^{-2\sqrt{(V_0/E_R)}}$ and local correlation $U \simeq E_R\sqrt{8/\pi}\, a_s k_L(V_0/E_R)^{3/4}$ [12]. Here, the recoil energy, $E_R = \hbar^2 k_L^2/2m$ is typically of order a few micro Kelvin. (k_L and V_0 are the wave-vector and intensity of the laser respectively). a_s is the s-wave scattering length, and the fermionic bandwidth, $W << E_R$.

Typically, $t/\hbar \simeq 1$ kHz and $U/\hbar \simeq 20-30$ kHz [13]. So $U/W \simeq O(2-3)$ in three dimensions. We envisage comparable values for the hoppings and U_{ab} in our case. This is precisely the intermediate to strong coupling regime of the $U_{ab} < 0$ "Hubbard"-like model that we study below. Focussing solely on possible Kondo-like physics, we use dynamical mean-field theory (DMFT) to study the model. Its reliability for understanding quasi-local collective Kondo coherence in correlated metals is already established [16]. Here, we show how a combination of experimental manipulations in optical lattices facilitates observation of the elusive CKL state.

Eq. (1) is a generalized *negative-U* Hubbard model (relabeling $a \to c_\uparrow, b \to c_\downarrow$) with "spin"-dependent hoppings, and a "magnetic field". Given the isomorphism between the $U > 0$ Hubbard model and the $U < 0$ Hubbard model in a magnetic field (seen by performing a particle-hole transformation), the possibility of the *charge* Kondo effect (CKE) involving pair-pseudospin-quenching by itinerance manifests itself (the pseudospin $\tau_i^\alpha = c_{ia}^\dagger \sigma_{ab}^\alpha c_{ib}$). On the other hand, lack of pseudospin SU(2) invariance of H (lowered to $Z_2 \times U(1)$) generically favors *ordered* ground states, either a charge-density wave (CDW) or a superfluid (SF). A CKL state can then appear as a *ground* state only upon quantum melting of these ordered states, or for model parameters such that $T_{CKL} > T_{CDW,SF}$.

To proceed, we "rotate" the a,b Fermions: writing $f_a = (ua+vb)/\sqrt{u^2+v^2}, f_b = (va-ub)/\sqrt{u^2+v^2}$ with $u = \sqrt{t_{aa}/(t_{aa}+t_{bb})}, v = \sqrt{t_{bb}/(t_{aa}+t_{bb})}$ on the parameter curve $t_{ab} = \sqrt{t_{aa}t_{bb}}$ leads to

$$H = t\sum_{<i,j>}(f_{ia}^\dagger f_{ja} + h.c) - U_{ab}\sum_i n_{ifa}n_{ifb} + \Delta\sum_i(f_{ia}^\dagger f_{ib} + h.c.) + \sum_{i,\alpha=a,b}\epsilon'_\alpha n_{i\alpha} \qquad (2)$$

where $t = (t_{aa}+t_{bb})$, $\Delta = \Delta_0 - uv$ and $\epsilon'_a = v^2\epsilon_a, \epsilon'_b = u^2\epsilon_b$. This is the $U < 0$ Falicov-Kimball model (FKM) with a *local* hybridization [17]. If Δ is tuned to zero, this reduces to the pure FKM. Since the FKM can be *exactly* solved in $D = \infty$ [18], we first solve H_{FKM} and then consider the effects of $t_{ab} \neq \sqrt{t_{aa}t_{bb}}, \Delta \neq 0$.

We observe that $[n_{ifb}, H_{FKM}] = 0$ for *each* i, implying a *local* U(1) invariance of H_{FKM}. This is *exactly* the condition for degenerate, resonant charge fluctuations between local configurations with $n_{f_b} = 0, 1$ to occur. Elitzur's theorem implies that both local excitonic ($\langle f_{ia}^\dagger f_{ib}\rangle$) and local pairing ($\langle f_{ia}^\dagger f_{ib}^\dagger\rangle$) averages rigorously vanish. Choosing the f_a-band density-of-states (DOS) to be a lorentzian with half-width W; i.e, $\rho_0(E) = W/\pi(E^2 + W^2)$, the f_a-band propagator within DMFT is *exactly* calculable as [4] $G_{f_a}(\omega) = \frac{1-n_{fb}}{\omega+iW} + \frac{n_{fb}}{\omega+U_{ab}+iW}$, where $n_{fb} = (1/N)\sum_i\langle f_{ib}^\dagger f_{ib}\rangle$ with N the number of lattice sites. Correspondingly, the f_b-Fermion propagator exhibits an infra-red singular, branch cut form, $G_{f_b}(\omega) \simeq \theta(\omega)|\omega|^{-(1-\eta)}$ at low energy. Here, $\eta = \tan^{-1}(U_{ab}/W)/\pi$ is the so-called s-wave phase shift of the f_a-band Fermions induced by the strong scattering term (U_{ab}) in H_{FKM}. The f_b-Fermion self-energy, $\Sigma_{f_b}(\omega) \simeq |\omega|^{1-\eta}$, implying that the FL quasiparticle residue $Z = 0$, and the symmetry-unbroken metallic phase is a non-FL metal. The underlying cause for non-FL behavior is explicit in the impurity limit of H_{FKM}, where recoil-less (infinitely heavy) f_b-fermions scatter the f_a-band fermions: it is the X-ray edge problem, where such an infra-red singularity is known to occur. This implies that the local *pairing* susceptibility (the analogue of the "excitonic" susceptibility for $U > 0$) also diverges in the infra-red, $\chi'_{f_a,f_b}(\omega) = \int dt e^{i\omega t}\langle f_{ia}^\dagger f_{ib}^\dagger(t); f_{ib}f_{ia}(0)\rangle \simeq |\omega|^{-(2\eta-\eta^2)}$: coupling of f_a-band fermions to this singular pair susceptibility then destroys FL theory. Within DMFT, this is an incoherent pair liquid state, where pair correlations diverge in the infra-red *without* SC order. We dub this state the incoherent Bose metal (IBM). The IBM is characterized by resonant charge fluctuations as described above, and alluded to in earlier work [10, 19]. The mixing of the degenerate single-particle and pair states reflect the superposition of the "emission" and "absorption" initial states of the (presently, attractive-U) X-ray edge problem. Since the f_b-fermions are rigorously localized, the "Fermi surface" now wholly corresponds to the dispersive f_a-band quasiparticles, with a smeared dispersion reflecting unusually broad pseudoparticles, even at $T = 0$. Depending on $(t_{aa}+t_{bb})/U_{ab}$ and along the curve $t_{ab} = \sqrt{t_{aa}t_{bb}}$, we generically find a *first order* transition near half-filling, where the total fermion density, n, jumps from $n_< = (n/2) + \frac{1}{\pi}\tan^{-1}(U_{ab}/2W)$ to $n_> = (3n/2) - \frac{1}{\pi}\tan^{-1}(U_{ab}/2W)$. For a bounded unperturbed DOS, numerical DMFT calculations give results closely paralleling those described above [20]. At a critical coupling, $g < g_c = (U_{ab}/2W)_c$, this line of first-order transitions ends in a second order *quantum critical* end point (QCEP) at $T = 0$ [21].

Away from $\Delta = 0$ or $t_{ab} = \sqrt{t_{aa}t_{bb}}$, the terms, $\Delta\sum_i(f_{ia}^\dagger f_{ib} + h.c.)$ (first case) or $(t_{aa}t_{bb} - t_{ab}^2)\sum_{<i,j>}(f_{ia}^\dagger f_{jb} + h.c)$ (second case) change the *local* U(1) invariance of H_{FKM} to a *global* U(1) invariance: only the *total* fermion number
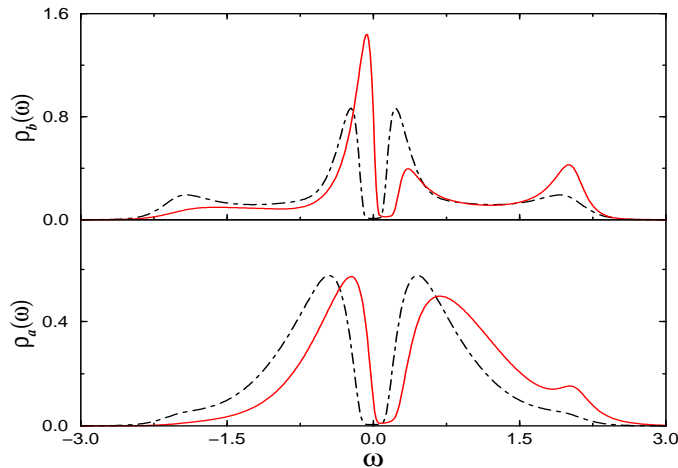
FIG. 1: Local spectral function of the $U_{ab} < 0$ periodic Anderson model within DMFT in the symmetric (black, dashed) and asymmetric (red, dotted) limits for the a (lower panel) and b Fermions (upper panel). Upon varying ϵ'_a, the charge Kondo insulator ($\epsilon'_a = 0$) melts into a charge-Kondo-FL state for $\epsilon'_a \neq 0$.

commutes with H, $[n_{f_a} + n_{f_b}, H] = 0$. This global symmetry can indeed be (and is) spontaneously broken, and, away from "half-filling", gives a *color* singlet superfluid (CSS) at low T, characterized by $\Delta_{ab} = \sum_i \epsilon^{ab} \langle f^\dagger_{ia} f^\dagger_{ib} \rangle > 0$ below $T_{SC} = f(U_{ab}/t, \Delta, n)$. This is readily checked by studying H within the generalized Hartree-Fock random phase approximation [6], which is valid now, since there is no local constraint for any $\Delta \neq 0$. The enhancement of the superconducting T_c in doped $PbTe$ and $SnTe$ [9] is now rationalized [11] as a manifestation of the proximity to the resonant charge fluctuation regime driven by strong quantum fluctuation between the two degenerate valence states, $n_{f_b} = 0, 1$, envisaged in the original prediction [10], for $\Delta \simeq 0$ as above. The respective transition temperatures can be computed from DMFT [22] over the full range of $|U_{ab}|/(t_{aa}, t_{bb}, t_{ab})$, and yields a *smooth* BCS-BEC crossover with increasing $|U_{ab}|$.

This implies that the charge Kondo screening can *never* produce a CKL *ground* state. However, *above* the ordering scales, relevance/irrelevance of the $f_a - f_b$ hybridization determines whether the IBM or a correlated FL metal will result. Consider the symmetry unbroken phase for $\Delta > 0$. The term $\Delta \sum_i (f^\dagger_{ia} f_{ib} + h.c)$ hybridizes the "infinitely heavy" (for $\Delta = 0$ above) f_b fermion with the f_a-band. This is precisely the $U_{ab} < 0$ periodic Anderson model (PAM) with a *local* hybridization between the "light" (a) and "heavy" (b) fermions. To access the physics in this regime, we solve this PAM within DMFT.

We choose a Gaussian unperturbed DOS for the a-fermions with $W/\hbar = 12\text{kHz}$ and $|U_{ab}|/\hbar = 20\text{kHz}$ as representative values. To solve the impurity problem of DMFT, we use the iterated perturbation theory (IPT) as an impurity solver. Prior work carried out for the $U > 0$ periodic Anderson model with additional $f - d$ interaction [23] has demonstrated its accuracy vis-a-vis the more "exact" QMC solver [22]: the IPT spectral functions closely match those extracted from QMC at considerably less numerical cost. Additionally, self-energies can be readily extracted from IPT in contrast to the difficulties encountered with QMC in this context. In Fig. 1, we show the results obtained within DMFT for the PAM for two cases: (i) $\epsilon'_a = 0$, and (ii) $\epsilon'_a \neq 0$. The first case has particle-hole symmetry, while the second does not. For a half-filled a-band, in case (i), we find a hybridization-induced Kondo insulator, where the hybridization gap opens up *after* local dynamical correlations have generated a charge-pseudospin quenched analogue of the Kondo screened state. The appearance of the high energy Hubbard bands is an explicit manifestation of relevance of local dynamical correlations. At lower energies, hybridization opens up a (renormalized) band insulating gap. Interestingly, this is a new type of insulating state, which we dub a *Charge Kondo Insulator* (CKI). Obviously, since $G_{ab}(\omega) = \langle f_a; f^\dagger_b \rangle$ is finite, this CKI state supports a finite excitonic polarization in the sense of Portengen *et al.* [24].

In case (ii), lack of particle-hole symmetry, i.e $\epsilon'_a \neq 0$, effectively leads to partially filled a, b bands, inducing a correlated metallic state within DMFT. In this case, the $T = 0$ spectral function of the b ("heavy") fermion shows a sharp quasiparticle resonance near $\omega = 0$, and examination of the self-energy shows that the metallic state is a renormalized Fermi liquid (FL). "Related signatures of the CKI to CKL transition are also clearly visible in the "conduction" (a) band DOS." Thus, DMFT yields a correlated FL metal due to dynamical quenching of the charge-

pseudospin degree of freedom by itinerance of the b-fermion (due to "hybridization", Δ, in Eq. (1)). Emergence of this charge-Kondo-FL (CKL) within DMFT for the $U_{ab} < 0$ PAM is understood as follows. In the impurity version of the lattice problem, we have an X-Ray edge problem with recoil (Eq. (2)) for $\Delta \neq 0$. Within DMFT, the infra-red singularities described above for $\Delta = 0$ are immediately cut off *below* a low energy scale associated with this recoil. Expressing the low-energy part of H in terms of collective, *bosonic* variables [25], this coherence scale, below which correlated FL behavior sets in, is estimated to be $k_B T_{coh} = Z \simeq \exp(U_{ab}^2 ln(\gamma)/(1 - \gamma^2))$ with $\gamma = m_{fa}/m_{fb}$, the mass ratios of the $f_{a,b}$ fermions. In our DMFT calculation, we find $Z \simeq 0.3$ from the real part of the self-energy (not shown). Hence, with $t_b = 0.3 t_a$, we obtain $T_{coh} \simeq 0.07 T_F$. With a slightly smaller value of U_{ab}/W, T_{coh} can be increased to $0.1 T_F$, which would lie in the lower range of temperatures observable in current experimental realizations [13]. Above T_{coh}, the irrelevance of the "hybridization" (Δ) implies that the system crosses over *smoothly* to the IBM phase found for $\Delta = 0$. Away from $t_{ab} = \sqrt{t_{aa}t_{bb}}$, the f_b-fermions once again acquire a finite mass, giving correlated FL behavior below T_{coh}, or broken symmetry ground states.

Only if $T_{coh} > T_{SC,CDW}$ will the CKL state be observable in a small window in parameter space. The above analysis suggests a way to observe the CKL state experimentally in optical lattices:

(a) force a mixture of two atomic states via a Feshbach resonance (creating attractive interaction),

(b) apply sequential, time-synchronized laser excitation to simulate the "hybridization", and

(c) "tune" Δ to relevance in the RG sense, by manipulating the one-body potential [15].

The change in the Fermi surface (FS) as the system is tuned across $t_{ab} = \sqrt{t_{aa}t_{bb}}$, $\Delta = 0$ could be detected by time-of-flight measurements [26] for $T > T_{SC,CDW}$. At $t_{ab}, \Delta = 0$, a single smeared FS, corresponding to itinerant f_a quasiparticles, should be seen, while, for $t_{ab} > \sqrt{t_{aa}t_{bb}}$ and/or $\Delta \neq 0$, *two* well-defined FS sheets (**k**-space eigenvalues of the free part of H in Eq. (1), corresponding to *coherent* quasiparticle propagation, should be observable. Since the self-energy is strictly *local* in DMFT, the FS will be unaffected by interactions, which then have the sole effect of critically (or overcritically) damping the FS. Given that the CKL state involves local, dynamical renormalization, designing systems where the actual, measured FS agrees well with that computed without interactions will show improved chances of experimentally observing the CKL state.

Recently, the quasiparticle dispersion and the spectral function of strongly interacting fermions exhibiting a BCS-BEC crossover in an optical trap has been experimentally deduced by Stewart *et al.* [27]. We suggest that similar experiments carried out below $T_{coh} \simeq 0.07 T_F$ should exhibit a relatively sharp low-energy peak in $A_k(\omega) = (-1/\pi)\text{Im}[1/(\omega - \epsilon_{ka} - \Sigma_a(\omega))]^{-1}$: this would constitute a characteristic signature of the CKL state. The CKI state in the symmetric limit above could also be probed using this technique: one should observe two (a and b bands) dispersive features below *and* above E_F, with a finite *hybridization* gap at E_F. At higher T currently accessible, we predict an incoherent pseudogap structure in $A_k(\omega)$ at low energy. Achieving lower T may be feasible in the near future: theoretical suggestions to do this by efficient shaping of the optical lattice profile is predicted to lower the entropy per particle by a factor of 10 [28].

Our study also permits drawing an analogy with the phase diagram of nuclear matter. In quantum chromodynamics (QCD), where *three* quarks form a *trion* bound state, a *baryon* in the $N = 3$ (color) theory, one also has, as a function of chemical potential, "color superfluid" (CSS) and high-T, quark matter phases. In the cold-atom setting, in a pioneering work, such a trion gas phase has theoretically been shown to be separated from a color superfluid phase by a *second order* transition [29]. Let us now study the possibility of realizing analogous physics in our case. Our model corresponds to the "$N = 2$" version of QCD, where the CSS ($p - p$ order) phase is separated from the excitonic ($p - h$ order) and incoherent IBM phases by a line of *first order* transitions, ending in a second order end-point, generically at finite T. Interestingly, the IBM phase has co-existing single-particle ("quark") and pair states at low-energy; it is an incoherent state where "quarks" and "pairs" continuously transmute into each other, their equilibrium fraction being determined by the ratio U_{ab}/t, Δ and T. The confinement-deconfinement transition, familiar in the QCD context, can be illuminated in our case by employing bosonization of the "impurity model" of DMFT. At $\Delta = 0$, the impurity version of H_{FKM} is readily *bosonized* [30] into a collection of radial bosonic models about the "impurity". This reads $L_0 = v_F \int [\Pi^2(r) + (\partial_r\phi(r))^2]dr$, where $v_F = 2ta$ is the carrier fermi velocity. For small $\delta = (t_{ab} - \sqrt{t_{aa}t_{bb}}) \neq 0 (< 0)$, the $f_a - f_b$ hybridization is small enough that the *two-particle* hopping terms (second order in δ) are more relevant. These terms are $H_{res} \simeq -(\delta^2/U_{ab}) \sum_{<i,j>} f_{ia}^\dagger f_{ib}^\dagger f_{jb} f_{ja}$, which can be decomposed in two ways in a Hartree-Fock decoupling (exact in $D = \infty$, and in the corresponding impurity model, this corresponds to two, $p - h$ and $p - p$ pairing fields), generating either excitonic (particle-hole) order (in the imbedded impurity problem of DMFT) with order parameter $\Delta_{ph} = \sum_i \langle f_{ia}^\dagger f_{ja} + f_{ib}^\dagger f_{jb} \rangle > 0$ or CSS (particle-particle) order, i.e., $\sum_i \epsilon^{ab} \langle f_{ia}^\dagger f_{ib}^\dagger \rangle > 0$. Bosonization then yields a *dual cosine* quantum sine Gordon (QsG) model:

$$L = L_0 - g_1 \int cos(\beta_1 \phi(r)) dr - g_2 \int cos(\beta_2 \theta(r)) dr \tag{3}$$

with $\beta_1 = \sqrt{8\pi K}$ and $\beta_2 = \sqrt{8\pi/K}$. $K = [1 - (U/2\pi v_F)]^{-1}$ in the weak coupling limit for our negative$-U$ model exactly at the FK point in the impurity model. A renormalization group analysis of L shows two different phases, depending upon the sign of $\delta g = (g_1 - g_2)$ [31]:

(i) when $g_1 > g_2$, the $\phi(r)$ field acquires a finite expectation value, giving $p - h$ excitonic order. In QCD lore, this translates into $\langle \bar{\psi}_\alpha \psi_\alpha \rangle > 0$, with $\alpha = a, b$, i.e, to a chiral *meson* phase, and,

(ii) when $g_2 > g_1$, the $\theta(r)$ field acquires a finite expectation value, giving CSS order. This corresponds to $\langle \psi_a \psi_b \rangle > 0$, i.e, to a color-singlet superfluid in QCD.

(iii) Interestingly, these phases are separated by a critical curve $g_1 = g_2$ or $K = 1$, where the QsG model becomes *self-dual*. This is exactly soluble, and the correlation functions are characterized by continuously varying, K-dependent exponents.

The above meson and CSS phases arise from the IBM (bosonic gaussian model) via Kosterlitz-Thouless transitions (driven by relevance of either of the cosine terms for $K < 1$ or $K > 1$), and are separated from each other by a line of first-order transitions, ending in a second-order endpoint at finite $T = T^*$. For $T > T^*$, *both* $\langle \bar{\psi}\psi \rangle, \langle \psi\psi \rangle = 0$, corresponding to "quark matter" in QCD [32]. Of course, this analogy is only suggestive: in our case the a, b particles have very different masses, in contrast with the small mass difference between u, d quarks in QCD. Nevertheless, the analogy is interesting, and generalization to $N = 3$ is possible. We will extend this qualitative analogy in more depth in future.

In conclusion, from a DMFT calculation, we propose that a combination of wilfully tunable Feshbach resonances *and* sequentially applied laser pulses in optical lattices to tune $U_{ab}/t_{ab}, \Delta$ through a desired range, could facilitate the observation of the elusive CKL state. Time-of-flight measurements can, by probing the "Fermi surface", provide valuable guidance on the experimental conditions favoring observation of the CKL state. Our analysis also shows interesting analogies with the "phase diagram" of nuclear matter. Extensions to *asymmetric* interaction models [14], allowing for realization of more exotic ordered phases and *anisotropic* CKL states within the DMFT approach, is also foreseen.

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[1] A. C. Hewson, *The Kondo Problem to Heavy Fermions*, (Cambridge University Press).

[2] D. L. Cox, Phys. Rev. Lett.**59**, 1240 (1987).

[3] L. Craco, *et al.*, Phys. Rev. Lett. **90**, 237203 (2003). O. Kolesnychenko, *et al.*, Phys. Rev. B**72**, 085456 (2005).

[4] Q. Si, *et al.* Phys. Rev. B**46**, 1261 (1992).

[5] C. M. Varma, Phys. Rev. Lett. **61** 2713 (1988).

[6] A. Taraphder, *et al.*, Phys. Rev. B **52** 1368 (1995).

[7] A. Taraphder, *et al.*, Int. Jour. Mod. Phys. B **10** 863 (1996) *and references therein.*

[8] J. A. Wilson and A. Zahrir, Rep. Prog. Phys. **60**, 941 (1997).

[9] Y. Matshushita, *et al.* Phys. Rev. Lett. **94**, 157002 (2006); I. Erickson, *et al.* arxiv cond-mat/0809.0924 (2008); arxiv cond-mat/0904.3359 (2009).

[10] A. Taraphder and P. Coleman, Phys. Rev. Lett. **66** 2814 (1991).

[11] M. Dzero and J. Schmalian, Phys. Rev. Lett. **94**, 157003 (2005).

[12] A. Georges, arXiv:cond-mat/0702122.

[13] R. Jördens, *et al.*, Nature **455**, 204 (2008).

[14] C. A Regal, *et al.*, Phys. Rev. Lett. **90**, 053201 (2003); J. Zhang, *et al.*, Phys. Rev. A. **70**, 030702 (2004); Q. Chen, *et al.*, Phys. Rep. **412**, 1 (2005).

[15] O. Mandel *et al.* Phys. Rev. Lett. **91**, 010407 (2003); O. Mandel, *et al.*, Nature (London) **425**, 937 (2003).

[16] G. Kotliar, *et al.*, Revs. Mod. Phys. **78**, 865 (2006).

[17] L. Craco, Phys. Rev. B **59**, 14837 (1999).

[18] U. Brandt and C. Mielsch, Z. Phys. B. **75**, 365 (1989); M. S. Laad, *et al.* Phys. Rev. Lett. **87**, 246402 (2001).

[19] D. Das and S. Doniach, Phys. Rev. B. **60**, 1261 (1999); P. Phillips and D. Dalidovich, Science **302**, 243 (2003).

[20] P. van Dongen, *et al.* Phys. Rev. B **64**, 195123 (2001).

[21] S. Watanabe *et al.*, Phys. Rev. Lett. **100**, 236401 (2008), for the $U_{ab} > 0$ case; we expect similar behavior for $U_{ab} < 0$ in the symmetry-unbroken phase within DMFT.

[22] M. Jarrel, H. Akhlaghpour and Th. Pruischke, Phys. Rev. Lett. **70**, 1670 ((1993); N. S. Vidhyadhiraja, *et al.* Europhys. Lett. **49**, 459 (2000).

[23] L. Craco, Phys. Rev. B **77**, 125122 (2008).

[24] T. Portengen, Th. Oštreich and L. J. Sham, Phys. Rev. B **54**, 17452 (1996).

[25] E. Müller-Hartmann, *et al.* Phys. Rev. B**3**, 1102 (1971).

[26] I. Bloch, *et al.*, Rev. Mod. Phys. **80**, 885 (2008).

[27] J. T. Stewart, J. P. Gaebler and D. S. Jin, Nature **454**, 744 (2008).

[28] J-S. Bernier *et al.*, Phys. Rev. A **79**, 061601(R) (2009).

[29] A. Rapp, *et al.*, Phys. Rev. Lett.**98**, 160405 (2007).

[30] K. D. Schotte, *et al.*, Phys. Rev. **182**, 479 (1969).

[31] A. Gogolin, A. Nersesyan and A. Tsvelik, in *Bosonization and Strongly Correlated Systems* (Cambridge Univ. Press, Cambridge, 1998).

[32] S. Hands, Contemp .Phys **42**, 209 (2001).